

Answer the following questions. "Neat and clear answers are appreciated"

Question No. 1:

- a) What is meant by thermal conductivity of a material. What are the possible factors which may affect the decision of choice of an insulating material for an engineering application.
- b) The construction of a furnace wall consists of two layers: 7.5 cm fire brick of thermal conductivity $1.1 \text{ W/m}^\circ\text{C}$ and 0.64 cm mild steel plate of thermal conductivity $39 \text{ W/m}^\circ\text{C}$. The temperature of the inside surface of the wall is kept constant at 920 K while the temperature of the surrounding air is 300 K . If the heat transfer coefficient from the outside surface of the wall is $68 \text{ W/m}^2 \text{ }^\circ\text{C}$, determine the heat transfer losses per unit area of the wall and the outside surface temperature.

Question No. 2:

- a) Heat is uniformly generated inside a solid circular rod by the rate of $q_v \text{ W/m}^3$. The length of the rod is long enough such that all of the generated heat is considered to diffuse to the outer surface of the rod in the radial direction. Starting from the general equation of heat conduction in cylindrical coordinates:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_v}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

deduce an expression for the temperature distribution inside the rod. Schematically draw this distribution and show that the maximum temperature inside the rod is expressed as the following:

$$T_{\max} = T_w + \frac{q_v R^2}{4k}$$

Where, T_w is the outer surface temperature of the rod

R & k are the radius and thermal conductivity of the rod respectively.

- b) An electric wire has a diameter of 10 mm , thermal conductivity of $144 \text{ W/m}^\circ\text{C}$ and electrical resistance of $0.1 \text{ } \Omega/\text{m}$. The wire is uniformly covered by an insulating layer of 3 mm thickness and $0.15 \text{ W/m}^\circ\text{C}$ thermal conductivity. The wire is used in an environment of temperature $30 \text{ }^\circ\text{C}$ and the heat transfer coefficient from the outside surface of the insulating layer is $86 \text{ W/m}^2 \text{ }^\circ\text{C}$. If the maximum operating temperature inside the wire should not exceed $150 \text{ }^\circ\text{C}$, what is the maximum safe electric current may flow through the wire?

With best wishes

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Q(1) b

Given:- Composite plane wall

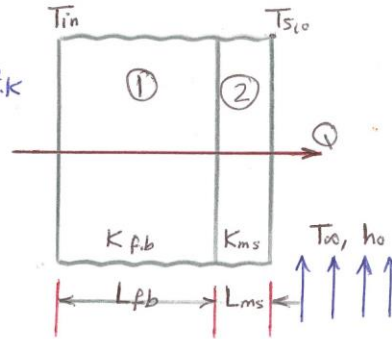
$$T_{in} = 920 \text{ K} \quad T_o = 300 \text{ K} \quad h_o = 68 \text{ W/m}^2\text{K}$$

$$L_{fb} = 0.075 \text{ m} \quad L_{ms} = 0.0064 \text{ m}$$

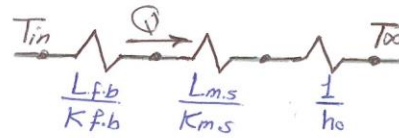
$$K_{fb} = 1.1 \text{ W/m.K} \quad K_{ms} = 39 \text{ W/m.K}$$

Req:- ① $q = \frac{Q}{A}$ ② $T_{s,10}$

Soln



$$q = \frac{Q}{A} = \frac{\Delta T}{\sum R_{th}}$$



$$\Delta T = T_{in} - T_o = 920 - 300 = 620 \text{ K}$$

$$\sum R_{th} = R_{cond①} + R_{cond②} + R_{conv}$$

$$= \frac{L_{fb}}{K_{fb}} + \frac{L_{ms}}{K_{ms}} + \frac{1}{h_o}$$

$$= \frac{0.075}{1.1} + \frac{0.0064}{39} + \frac{1}{68} = 0.08305 \frac{\text{K.m}^2}{\text{W}}$$

$$q = \frac{\Delta T}{\sum R_{th}} = \frac{620}{0.08305} = 7465.2 \text{ W/m}^2$$

for steady state heat transfer $\Rightarrow q$ is the same through all layers

$$q = \frac{T_{in} - T_o}{\frac{L_{fb}}{K_{fb}} + \frac{L_{ms}}{K_{ms}} + \frac{1}{h_o}} = \frac{T_{in} - T_{s,10}}{\frac{L_{fb}}{K_{fb}} + \frac{L_{ms}}{K_{ms}}}$$

$$T_{s,10} = T_{in} - q \left(\frac{L_{fb}}{K_{fb}} + \frac{L_{ms}}{K_{ms}} \right)$$

$$= 920 - 7465.2 \left(\frac{0.075}{1.1} + \frac{0.0064}{39} \right)$$

$$T_{s,10} = 920 - 510.216 \Rightarrow \boxed{T_{s,10} = 409.784 \text{ K}}$$

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assumption

① one-direction H.T ② steady state H.T

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = -\frac{q_v}{k} \xrightarrow{*r} r \frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r} = -\frac{q_v \cdot r}{k}$$

$$\frac{\partial}{\partial r} \left(r \cdot \frac{\partial T}{\partial r} \right) = -\frac{q_v \cdot r}{k} \xrightarrow{\int} r \cdot \frac{dT}{dr} = -\frac{q_v \cdot r^2}{2k} + C_1$$

$$\frac{dT}{dr} = -\frac{q_v \cdot r}{2k} + \frac{C_1}{r} \rightarrow \text{I} \xrightarrow{\int}$$

$$T = -\frac{q_v \cdot r^2}{4k} + C_1 \ln r + C_2 \rightarrow \text{II}$$

B. conditions

$$\text{At } r=0 \Rightarrow \left(\frac{dT}{dr} = 0 ; T = T_{\max} ; T \text{ is limited} \right) \Rightarrow (C_1 = 0)$$

$$\text{At } r=R \Rightarrow T = T_w \Rightarrow T_w = -\frac{q_v \cdot R^2}{4k} + C_2 \Rightarrow C_2 = T_w + \frac{q_v \cdot R^2}{4k}$$

$$T = -\frac{q_v \cdot r^2}{4k} + T_w + \frac{q_v \cdot R^2}{4k} \Rightarrow T = T_w + \frac{q_v \cdot R^2}{4k} \left(1 - \left(\frac{r}{R} \right)^2 \right)$$

an expression for Temperature distribution

At $r=0 \Rightarrow T = T_{\max} \rightarrow$ sub in eqn (III) we obtain

$$T_{\max} = T_w + \frac{q_v \cdot R^2}{4k}$$

